Energy Flow in Joined Plate Girders in Low, Medium and High Frequency Range

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Analysis of structural vibrations in medium frequency range achieved a lot of attention in recent years. The necessity of special prediction tools have been widely emphasized. This is because of the drawbacks of finite element method and statistical energy analysis. In the present paper the problem of modelling of steel plate girders will be considered. The low, medium and high frequency regions are addressed. The results of finite element and statistical energy analysis are compared with experimental results. This comparison is made based on the mode count, input and transfer mobilities. The comparison of the mode count shows good agreement between finite element solution, experimental results and asymptotic analytical expressions used widely in statistical energy analysis. In the comparison of input mobility the differences are more visible, and this influences also the power flow calculations. The issue of modelling damping is brought forward and results of analysis with different damping values are compared. Finally, the conclusions regarding the preferred modelling scheme are presented.

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1. INTRODUCTION

In this paper the analysis of a T-joint made of steel plate girders will be presented. Plate girders are often used as components of steel railway bridges, and the outcomes of the recent research aim in analysing the whole bridge structure. The model consisting of three girders have been chosen to include a junction in the analysis.

The static and dynamic analysis of structures are often performed using finite element method (FEM) [12, 1]. However, FEM based analysis are restricted to low frequency ranges. This is because the adequate representation of deformation of the structure in high frequency require high number of degrees of freedom (DOF). The highest frequency that can be analysed depends on the structural complexity and the software/hardware used. Additionally, the deterministic approach like FEM fails in high frequency region, where the uncertainties in geometrical or physical properties cause major changes in system response to loading. Therefore the idealistic model described by FEM can describe the behaviour of the real structure only to certain extend.

In contrary, statistical energy analysis (SEA) is widely used in analysis of high frequency vibrations of the structures [9, 3, 8]. It describes the average energy of the subsystems, which are parts of the structure, when subjected to excitation, based on overall properties. The energy distribution among subsystems is setup

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based on the assumption that the power flow between subsystems is proportional to the difference between their modal energies and the coupling loss factor (CLF). In this way the real structure properties are taken into consideration as the average properties of the ensemble of possible properties this structure could have. The SEA predictions are limited at low frequencies because the response of the structure is dominated by single resonances and the non-resonant behaviour is of importance. One of the SEA assumptions is that there is sufficient number of natural frequencies within frequency band, so that the mode-to-mode coupling is present. Therefore, the method is limited to the frequency ranges where the resonances overlap, and the response is smooth enough, to be well described by its frequency average [5].

In the previous studies on the power flow between coupled plates Simmons [13] considered the structures forming L and H shape with two and five plates in assembly, respectively. He used finite element method to estimate the energy ratios between plates at discrete frequencies between 10 Hz and 2 kHz and compared the results with experimental ones. He reported discrepancies at individual frequencies when comparing FEM results and experimental results. Additionally, the difficulties in modelling damping was brought forward by the author. When the damping values derived from mobility measurements on the structure were used in FEM predictions, the discrepancy between measurements and predictions increased.

The possibility of deriving coupling loss factors for coupled plates by FEM simulations was also presented by Fredo, [6]. The comparison between FEM results and experiment was based on input point mobilities. In both of the previously mentioned references the prospect plates were used in experiment. In present studies the steel model is examined.

The transmission of structure-borne sound at low frequencies was also studied in [2, 15]. These two papers present the comparison between SEA, FEM, and experimental results for brick walls in frequency below 1 kHz. In [10] the energy flow models based on FEM analysis are presented for the three-plate structure forming a corner. The authors conclude that use of numerically efficient energy flow method based on FEM overcomes two of SEA drawback, namely it allows to analyse strongly coupled subsystems and takes into consideration mode-to-mode interaction.

Both FEM and SEA will be used to predict the response of the T-joint of plate girders under excitation. The results will be verified by comparing with the results of experiment. The comparison between FEM and SEA results is used quite often, and usually very complex FEM model is used in analysis as a benchmark to validate SEA predictions. This is usually done at small objects, where the FEM model can be used up to high frequencies.

The comparison between analytical and experimental results will be made based on the following criteria: mode count, input mobilities and power flow considerations. The mode count is a very important parameter for SEA, because it represents the number of resonant modes, that can receive and store energy [9]. It has additional advantage, namely it is reasonably simple to obtain and compare. The input mobility will be used in comparison for two reasons. Firstly, the spatial variation of the input mobility gives insight into the properties of the structure in different frequency ranges. Secondly, the input mobility determines the input power due to applied loading, and thus determines the level of response. Finally, the power
flow will be considered. This aims in comparing the transfer mobilities averaged over excitation points and receiving points. This spatial averaging is limited to few points on the structure only, and the influence of that fact will be discussed.

2. NUMERICAL MODELS

The numerical calculations require that the system with infinitely many degrees of freedom is approximated by the one with finite set of degrees of freedom. This can be achieved, for example, by finite element decomposition of the continuous domain in FEM or partition to subsystems in SEA.

2.1. FINITE ELEMENT MODEL

The standard commercial code (ABAQUS) [11] was used in FEM analysis. The finite element model consists of thin four node plate elements (S4R5). According to [4] the bending wavelength of a 3 mm thick plate at 7 kHz is approximately 60 mm. The maximum length of the element edge is 10 mm which gives the ratio of 6 elements per wavelength. Figure 1 presents a fragment of the finite element mesh. Some of the nodes have been chosen and labelled for further reference. These nodes lie at the positions of the measurement points. The global coordinate system will also be used as a reference in further description. The boundary conditions used

![Figure 1. Fragment of the numerical model with marked measurement points and global coordinate system.](image)

in FEM calculations are approximating the condition of soft elastic support, which is used in experiment. Therefore the three-dimensional springs are inserted in the
supported corners of the model. By modifying the stiffness of spring elements one may update the rigid-body modes of the model [7].

Two damping models were used: the commonly used value of modal damping ratio for steel structures $\zeta = 0.005$ (corresponding to the damping loss factor $\eta = 0.001$) for all modes and the experimentally obtained value. The experimental values of damping averaged within octave bands are summarised in Table 1.

2.2. SEA MODEL

Commercially available package (AutoSEA-2) was used to perform the SEA analysis. Fifteen plate subsystems were defined, see Figure 2. Line and point junctions were defined among the plates. The coupling loss factors are derived by the package based on geometry and physical properties of the elements. The same two models of damping as in the FEM were used in SEA. There SEA results depend strongly on the damping model used, see results in later sections.
3. EXPERIMENTAL SETUP

The experiment was performed on the welded model of the T-joint. The model was supported on soft rubber pads. An excitation pulse was applied by a hammer with built-in force transducer.

The acceleration transducer was used to measure the response in a single point. A two-channel analyser was used to record the force input signal and acceleration output signal. To obtain similar value of the impact force in subsequent measurements the hammer was attached to the pendulum.

The sampling rate was 38 kHz and the signal was recorded for half of a second. The hammer tip was chosen in such manner that the maximum frequency excited by the hammer was below the frequency of logging the data, and therefore no signal filtering was necessary. The maximum frequency excited by the hammer was approximately 8 kHz. The measurements are repeated ten times at each pair of points. Based on the set of ten measurements the mean value of non-calibrated signal is calculated. The fast Fourier transform (FFT) was performed on averaged data to reduce the influence of signal noise. The resulting frequency response function had to be calibrated to physical units to yield acceleration and multiplied by a \((i\omega)^{-1}\) to obtain the mobility.

The critical damping ratio was measured from the Nyquist plot of mobility in the vicinity of resonances. The results were then averaged in octave bands and used in both FEM and SEA.
Figure 4. Mode count for plate subsystems, a) infinite plate approximation, b) rectangular plate with approximated boundary conditions; · · ·, bending modes of flanges; - - - - , bending modes of webs; ---, all in-plane modes.

4. EVALUATION OF MODE COUNT

The first criterion used in comparison of experimental data and analytical model is the mode count (number of modes $N(\omega)$ or $N(f)$ below a certain circular frequency $\omega$ or frequency $f$). This quantity is crucial for the behaviour of the structure. In the following it should be quantified if the results of modelling the structure as the set of plates correspond to the results obtained from measurements.

In calculating the theoretical mode count modal summation approach was applied [9]. The modal summation approach assumes that the overall number of modes in built-up structure is well approximated by the number of modes in its structural components, or subsystems. According to the modal summation approach, several subsystems were distinguished:

- bending modes of each of the webs (simply supported plate),
- bending modes of each of the half-flanges (cantilevered plate),
- in-plane modes of webs and flanges (longitudinal and shear).

The approximate mode count for each of the subsystems can be calculated by considering the allowed mode shapes and the dispersion relations for the free waves in the subsystem [9]. However, some of the subsystems require deeper consideration.

When considering the mode count of the flanges, care has to be taken because of the geometry of the flange. The length-to-width aspect ratio (more than 20) does not correspond well to the expressions for rectangular plates, and therefore, the exact expressions for the natural frequencies of the cantilevered plate would be more appropriate in this case. Figure 4 presents results of the mode count for plate
subsystems, i.e. webs and flanges. Part a) of the figure presents the mode count derived from infinite plate approximation (i.e. mode count for the rectangular part of infinite plate of the same area as the one of the actual plate). Part b) shows the mode count derived from the considerations of simply-supported plates (for webs) and cantilevered plates (for flanges). The difference between mode counts from these two methods for webs (dashed line) is hardly visible, whereas for the flanges (continuous line) the differences are much stronger. Due to the mentioned aspect ratio, only the first bending mode cuts off in the 'short' direction in the frequency of interest (at approximately 4200 Hz). In the case of webs the cut-off frequencies of particular mode groups are visible at approximately 700 Hz, 2000 Hz and 3800 Hz. Both results are also included in Figure 5 for sake of comparison with other results (thin continuous and dotted line).

An additional factor influencing the distribution of natural frequencies, and therefore mode count, arises from the boundary conditions. In the evaluation of the mode count for each of the subsystems the boundary conditions are assumed either free, or clamped, whereas the real boundary conditions of the element in the built-up structure can be different. In fact the actual boundary conditions may also vary with frequency, and therefore, the mode count obtained from the modal summation approach is only the rough approximation. The actual natural frequencies may differ considerably from the ones obtained by this approach, and so the mode count at certain frequency would differ.

To check the accuracy of the mode summation procedure the derived mode count was compared with the results of FEM-based eigenvalue analysis and measurement
results. In the FEM model it is necessary to ensure that the multi-degree-of-freedom description is accurate to describe the actual motions of the structure. Typically, 6-8 elements per wavelength are required to ensure the appropriate outputs. This condition is met below 6 kHz for the modeled considered. The results of the FEM analysis are presented in Figure 5 (thick dashed line).

To estimate the experimental mode count the resonant peaks of a few frequency response functions were counted. According to [9] this method can be used providing the modes are distinguishable, i.e. the frequency spacing $\delta f$ being bigger than the modal bandwidth $\pi \eta f/2$. If the modes are not spaced enough, then the results based on counting resonant peaks would underestimate the actual value.

The comparison between the mode counts for the overall structure is presented in Figure 5. The experimental results are obtainable only below 6 kHz, and even then there is a big chance that the repeating frequencies were not distinguished. There are two SEA curves: 'SEA-inf.' curve was obtained by using asymptotic expressions from infinite plate theory; 'SEA' curve is the one that takes into consideration the distribution of natural frequencies of rectangular plate.

5. INPUT MOBILITY

This section deals with the second quantity used in the comparison, the input mobility. The input mobility is defined as the complex harmonic velocity due to a unit harmonic force at the driving point. The experimental mobility can be calculated from the measured acceleration (the ratio between the amplitude of acceleration vector and amplitude of force vector in given direction) which in turn is the Fourier transform of the time signal. The numerical mobility is calculated at discrete frequencies from steady-state response to the harmonic point load. The frequency points at which the response is calculated are determined after modal decomposition, and are biased towards the natural frequencies.

5.1. INPUT MOBILITY OF THE FLANGE

Because of the similarity of the dimensions, the input mobility of the flanges should not differ considerably from one another, and therefore only one flange, of the vertical beam was tested.

According to [14] the average mobility of the finite plate should equal the mobility of infinite plate of the same thickness and material properties. The mobility of the infinite plate is given by [4]:

$$Y_{in} = Z_{in}^{-1} = 2.3 c_L \rho h^2$$  \hspace{1cm} (1)

for the infinite plate excited at the centre and:

$$Y_{in} = Z_{in}^{-1} = c_L \rho h^2$$  \hspace{1cm} (2)

for the semi-infinite plate excited at the edge; where $c_L$ is the velocity of the longitudinal wave, $\rho$ is the mass density of the material of the plate and $h$ is the thickness of the plate.
Figure 6. Magnitude of input mobility at point 2 at flange 3; --, numerical (FEM) results; - - -, experimental results.

Numerical mobilities are measured at five nodes of the FE model subjected to the unit harmonic force input. Four of these positions were repeated for measurements on the flange. The positions of measurement points are shown in Figure 1. Figure 6 presents the magnitude of input mobility at point 2 on the flange 3 measured and predicted by finite element calculations. The point force acts in the direction normal to the plate.

The agreement in terms of natural frequencies is reasonable, but the value of the peaks differs considerably. Further comparison will be presented in terms of one-third octave band averages instead of narrow-band ones. Figure 7a presents the spatial variation in the numerical results of input mobilities. The spatial variation for the experimental results, is presented in Figure 7b.

The measurements were taken at the same points on the flange as in the numerical calculations, so the results can be easily comparable. The comparison of point mobilities shows clearly the differences between numerical and experimental results. On the other hand, the similar tendencies of the curves are visible. Point 4 lies in the vicinity of the web, and therefore the mobility is much lower than of the two other points. This is because the mobility is dominated by the in-plane stiffness of the web. In contrary, the mobility at two points close to the edge (point 2 and 5), approach higher value at high frequency. At low frequencies the measured mobility of the girder is dominated by the stiff-body motion and therefore the mobility depends mainly on the vertical coordinate of the point. The FEM model accounts for the stiff-body modes (by introducing the spring support), but the resonant peak is narrow. In any case, the response of the system below 150 Hz is of minor
importance in the current consideration.

The comparison of results for the spatial and frequency average input mobility of the flange is presented in Figure 8. The plot presents theoretical mobility for plate excited at the edge and at the centre, as calculated in equation 1 and 2; average numerical mobility and average experimental mobility. The average mobility approaches the one of semi-infinite plate above 1.5 kHz.

5.2. INPUT MOBILITY OF THE WEB

The test method applied to the web of vertical beam was very similar to the one applied to the flange. Measurement points are marked in Figure 1. The points were chosen on one half of the web only, because of the symmetry. The frequency averaged results of numerical calculations are presented in Figure 9a; the experimental results are presented in Figure 9b. In low frequencies the mobility is dominated by the first bending mode and the first torsional mode of the vertical beam, see Figure 10. Therefore the response below 70 Hz is proportional to the vertical coordinate, and above that frequency there is additional contribution from the distance of the point from the central line of the beam. This is mostly visible in results for point 4, that lies close to the central line of the web, and whose response drops above 70 Hz. The next visible increase in the mobility coincides with the second bending mode, second torsional mode and second bending mode coupled with the first out-of-plane mode of the web of the vertical beam, see Figure 11. All these modes fall into the frequency band from 170 Hz to 350 Hz. Above 350 Hz there are no more visible peaks in the mobility, due to the high number of modes, so that the averaged value
approaches the one for infinite plate. This is also visible in the spatially-averaged values presented in Figure 12.

6. POWER FLOW

The next step in the analysis of the model is the comparison of the predictions of the energy distribution among subsystems with measured values. The predictions are made in two different ways. The first, based on the results of FEM calculations, the second is based on SEA calculation. The spatial average of the mechanical energy of harmonic out-of-plane vibrations of the subsystem is equal to the maximum of spatial average of the kinetic energy, i.e.

\[ \tilde{E} = \frac{1}{2} m_s (\langle \dot{V}_s^2 \rangle) \]  

(3)

where \( m_s \) is the mass of the subsystem \( s \), \( \langle \dot{V}_s^2 \rangle \) is the spatial average of the squared velocity amplitude of the harmonic vibrations and the \( \langle \rangle \) signifies the spatially averaged variable.

For the FEM-based predictions the averaging of squared velocity was performed over the same points as in the measurements, so that the averaging should have similar impact on the results as experimental data.

The results are presented in two different forms. The first is the energy of the subsystem normalized to the unit force input. The second is the energy normalized to the unit power input. The unit force input is simply a harmonic force with the
unit amplitude, i.e. $F(i\omega) = 1$. The power input by a harmonic point force is given by [4]:

$$W_{in} = \Re(F e^{i\omega t} V e^{i\omega t + \phi})$$

(4)

where $F$ and $V$ are the complex amplitudes of force and velocity and $\phi$ is the phase shift between them. The time-average input power is given as an integral of the input power over the period of the excitation process, so that the time-average input power is given as:

$$\dot{W}_{in} \frac{1}{2} \Re(FV^*) = \frac{1}{2} |F|^2 \Re(Y_{in})$$

(5)

where the asterisk denotes complex conjugate, $Y_{in}$ is the input mobility at the point of excitation. Again, the spatial averaging is required and it is performed over the measurement points.

It will be useful now to rearrange the expressions (3) and (5) so they are given in terms of input and transfer mobilities. For the normalization with unit force input, the energy in subsystem $s$ due to the excitation acting on subsystem $t$ is given as

$$\frac{\dot{E}_s}{F_t} = \frac{1}{2} m_s \frac{(|\dot{V}_t^2|)}{F_t} = \frac{1}{2} m_s (|\dot{Y}_{ts}|)$$

(6)

Figure 9. Spatial variation of input mobility of web 3, a) numerical results, b) experimental results; $- - -$, point 2; $- - -$, point 3; $- - -$, point 3; ...., point 5.
where the spatial averaging over the transfer mobility $Y_{st}$ has to be performed. For the normalization to unit power input, the expression will have the form:

$$ \tilde{E}_s = m_s \left( \frac{|Y_{st}^2|}{F_i^2 R(t)} \right) = m_s \frac{|Y_{st}^2|}{R(t)} $$

The results obtained by different methods are compared in the following. For each transmission path two different results are presented: one for the unit force

Figure 10. First bending mode (58.7 Hz) and first torsional mode (82 Hz) of the vertical beam.

Figure 11. Second bending mode (175 Hz), second torsional mode (234 Hz), and coupled bending and out-of-plane web modes of the vertical beam.
input, the other for the unit power input. The energy of web 1 when the unit force input is applied to flange 3, is presented in Figure 13. The SEA prediction using damping factor $\eta = 0.001$ overestimates the response level at all frequencies. If the measured values of the damping are used, the overestimation of SEA is at the level of 7-10 dB in frequencies above 350 Hz. The results of FEM with damping $\eta = 0.001$ agree very well with the experimental results above 200 Hz. Below 200 Hz there are remarkable differences between these results. This, however is a region where the rigid-body modes dominate the response. If the structure under consideration had real boundary conditions, these differences would probably vanish.

The improvement of the predictions can be achieved by using the unit power input instead of unit force input. This reduces the influence of input mobility in SEA predictions. The energy of web 1 normalized to unit power input applied to flange 3 is presented in figure 14. There is a better agreement between experimental results and SEA, and also the FEM results. The latter underestimate the response below 200 Hz.

Now, let us consider the energy flow from web to web. This should be a better case for SEA, since both the excited and receiving subsystems have a high number of modes. Figure 15 presents the comparison for the case of the force excitation. The SEA with damping ratio $\eta = 0.001$ overestimates the energy of approximately 10 dB. The FEM results with the same damping model agree better above 200 Hz. However, between 100 and 200 Hz the FEM the difference reaches 40 dB. This is due to the lack of resonances. Also, in the region below 50 Hz, where the stiff-body modes are present, the FEM gives too low prediction. If the damping model from
Figure 13. Energy of web 1 when the unit force input is applied to flange 3: —, experimental results; - - - , SEA, $\eta = 0.001$; - - - - , SEA, $\eta$ measured; - - - - - , FEM, $\eta = 0.001$; - - - - - - , FEM, $\eta$ measured.

Figure 14. Energy of web 1 when the unit power input is applied to flange 3: —, experimental results; - - - , SEA, $\eta = 0.001$; - - - - , SEA, $\eta$ measured; - - - - - , FEM, $\eta = 0.001$; - - - - - - , FEM, $\eta$ measured.

measurements is used, the response is underestimated by both prediction methods. It can be seen that the use of unit power input improves the agreement between
Figure 15. Energy of web 1 when the unit force input is applied to web 3: ——, experimental results; •••, SEA, $\eta = 0.001$; ••••, SEA, $\eta$ measured; •••••, FEM, $\eta = 0.001$; ••••••, FEM, $\eta$ measured.

Figure 16. Energy of web 1 when the unit power input is applied to web 3: ——, experimental results; •••, SEA, $\eta = 0.001$; ••••, SEA, $\eta$ measured; •••••, FEM, $\eta = 0.001$; ••••••, FEM, $\eta$ measured.
FEM and measurements, see Figure 16. The SEA results are in similar agreement that in the case of force input.

Finally, the energy in the driven web is considered. Figures 17, and 18 present the energy of web 3 when the unit force input and the unit power input is applied to web 3, respectively. Again, the agreement is better in the latter case, mainly for the FEM results. Also, the use of low damping value gives better results than the use of the experimentally obtained model.

7. CONCLUSIONS

Dynamic behaviour of plate girders in the medium and high frequency region is well modelled by the finite element model made up of plate elements. The numerical results converge well to experimental results for both input mobility and power flow. At low frequencies the FEM model suffices from the definition of the support. It is believed, this problem should not be present in the analysis of real structure, where the free boundary conditions are rather seldom.

Both numerical techniques are very sensitive to the definition of damping. If the low value of damping is assumed, than the SEA results generally overestimate the measured one, and the FEM results agree with the measured ones. The use of measured damping values in the calculations improves the performance of SEA, but the FEM predictions underestimate the response.

The high value of damping, mainly in low frequencies, is due to the damping of the rubber paths. Therefore, when considering the energy flow trough supported
Figure 18. Energy of web 3 when the unit power input is applied to web 3: ----, experimental results; - - - - , SEA, $\eta = 0.001$; - - - , SEA, $\eta$ measured; - - - - , FEM, $\eta = 0.001$; - - - - , FEM, $\eta$ measured.

flanges, the experimental value of damping is preferable. In contrary, for the web-to-web flows, the low damping of the girder should be used.

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